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KINEMATIC ANALYSIS OF SPATIAL SLAVE MECHANISMS FOR MANIPULATORS BY THE MATRIX METHOD

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ABSTRACT. The use of spatial mechanisms with many degrees of freedom as manipulator slave mechanisms is discussed. An analysis of these mechanisms by the matrix of 3rd order method is presented.

The recent development of a number of new branches of engineering requires 230 the employment of manipulators [1]. Spatial mechanisms with many degrees of freedom are used as the slave mechanisms in manipulators. A natural method of kinematic analysis of such mechanisms is the matrix method, which permits a more complete investigation to be made of their kinematics, as well as to order and reduce computations and thus facilitate the use of computer technology. Some of the first papers on the application of the matrix calculation to mechanisms were references [2, 3]. Reference [4] is devoted to the use of matrices with complex elements to analyze spatial mechanisms. Homogeneous coordinates, and matrices of 4th order were used successfully to analyze spatial gearing and spatial mechanisms in references [5, 6].

This paper analyzes the kinematics of spatial mechanisms for manipulators by using the matrix of 3rd order method. This method, as distinguished from known matrix methods for analyzing spatial mechanisms, is based on the simultaneous use of two matrix equations to transform the coordinates of a point and vector, and thus manage with matrices of 3rd order.

Let us establish the connection between the generalized coordinates characterizing the movement of the solid transported, and the generalized coordinates determining the position of the slave.

Let us examine first the slave without spherical pairs in Figure 1 (the mechanism of the "Mascot-1" manipulator [1]).

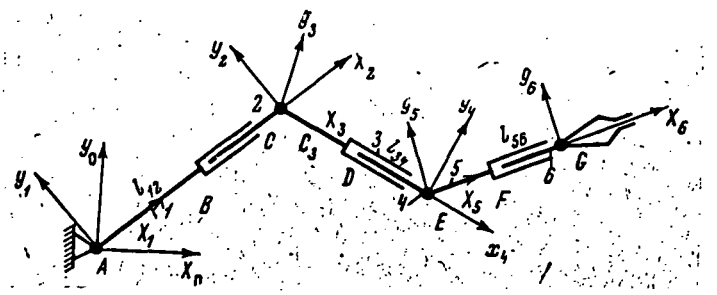

Figure 1.

Let us associate a system of coordinates with each link in the mechanism. 31
Let us associate the $X_0Y_0Z_0$ system with the support, placing the origin at A, and let us direct the Z_0 axis along the axis of pair A. Let us associate the $X_1Y_1Z_1$ system with link 1, placing the origin at A, and let us direct the X_1 axis along the axis of link 1, and the Z_1 axis parallel to the Z_0 axis. Let us associate the $X_2Y_2Z_2$ system with link 2, placing the origin at C, and let us direct the X_2 axis along the axis of link 2, and the Z_2 axis along the axis of turning pair C. Let us associate the $X_3Y_3Z_3$ system with link 3, placing the origin at C, and directing the X_3 axis along the axis of link 3, and the Z_3 axis along the axis of turning pair C. Let us asscoiate the $X_4Y_4Z_4$ system with link 4, placing the origin at E, and directing the X_4 axis along the axis of link 4, and the Z_4 axis along the axis of turning pair E. Let us associate the $X_5Y_5Z_5$ system with link 5, placing the origin at E, and directing the X_5 axis along the axis of link 5, and the Z_5 axis along the axis of turning pair E. And finally, let us associate the $X_6Y_6Z_6$ system with link 6, placing the origin at 6, and let us direct the X_6 axis along the axis of link 6, and the Z_6 axis along the axis of rotation of the tongs of pair G.

Let the coordinates of the pole of an arbitrary point, O, in the $X_6Y_6Z_6$ system rigidly coupled to the tongs link be equal to X_6, Y_6, Z_6. Let us write the coordinates of this same point in the $X_5Y_5Z_5$ system, using the matrix equation

$$X_5 = M_{56}X_6 + L_6, \tag{1}$$

where

X_5, X_6 are, respectively, the columns of the coordinates of point O in the $X_5Y_5X_5$ and $X_6Y_6Z_6$ systems;

M_{56} is the matrix of rotation for the transition from the $X_6Y_6Z_6$ system to the $X_5Y_5Z_5$ system;

L_6 is the parallel transfer matrice for the transition from the $X_6Y_6Z_6$ system to the $X_5Y_5Z_5$ system.

The $X_5Y_5Z_5$ system can be combined with the $X_6Y_6Z_6$ system by rotation about the X_5 axis through angle α_{56}, and by transfer along the X_5 axis by l_{56}, so the matrices have the forms

$$M_{56} = \begin{Vmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_{56} & -\sin\alpha_{56} \\ 0 & \sin\alpha_{56} & \cos\alpha_{56} \end{Vmatrix}, \tag{2}$$

$$L_{56} = \begin{Vmatrix} l_{56} \\ 0 \\ 0 \end{Vmatrix} \tag{3}$$

The $X_4Y_4Z_4$ system can be combined with the $X_5Y_5Z_5$ system by rotation about the Z_4 axis through angle γ_{45}.

Now the rotation matrix will have the following form for transition from the $X_5Y_5Z_5$ system to the $X_4Y_4Z_4$ system.

$$M_{45} = \begin{Vmatrix} \cos\gamma_{45} & -\sin\gamma_{45} & 0 \\ \sin\gamma_{45} & \cos\gamma_{45} & 0 \\ 0 & 0 & 1 \end{Vmatrix} \tag{4}$$

The matrix equation for the transition to the $X_4Y_4Z_4$ system is

$$X_4 = M_{54}(M_{56}X_6 + L_{56}), \tag{5}$$

where

X_4 is the column of the coordinates of point O in the $X_4Y_4Z_4$ system.

The $X_3Y_3Z_3$ system can be combined with the $X_4Y_4Z_4$ system by transfer along the X_3 axis by the magnitude l_{34} and by rotation about the X_3 axis through angle α_{34}. The rotation matrix for the transition from the $X_4Y_4Z_4$ system to the $X_3Y_3Z_3$ system is /32

$$M_{34} = \begin{Vmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_{34} & -\sin\alpha_{34} \\ 0 & \sin\alpha_{34} & \cos\alpha_{34} \end{Vmatrix} \tag{6}$$

The transfer matrix is

$$\begin{Vmatrix} l_{34} \\ 0 \\ 0 \end{Vmatrix} \tag{7}$$

The matrix equation for the transition to the $X_3Y_3Z_3$ system is

$$X_3 = M_{34}M_{54}(M_{56}X_6 + L_{56}) + L_{34}, \tag{8}$$

where

X_3 is the column of the coordinates of point O in the $X_3Y_3Z_3$ system.

The $X_2Y_2Z_2$ system can be combined with the $X_3Y_3Z_3$ system by rotation about the Z_2 axis through angle γ_{23}. The rotation matrix for the transition from the $X_3Y_3Z_3$ system to the $X_2Y_2Z_2$ system has the form

$$M_{23} = \begin{Vmatrix} \cos\gamma_{23} & -\sin\gamma_{23} & 0 \\ \sin\gamma_{23} & \cos\gamma_{23} & 0 \\ 0 & 0 & 1 \end{Vmatrix}. \tag{9}$$

The matrix equation for the transition to the $X_2Y_2Z_2$ system has the form

$$X_2 = M_{23}[M_{34}M_{45}(M_{56}X_6 + L_{56}) + L_{34}], \tag{10}$$

where

X_2 is the column of the coordinates of point 0 in the $X_2Y_2Z_2$ system.

The $X_1Y_1Z_1$ system can be combined with the $X_2Y_2Z_2$ system by transfer along the X_1 axis by the quantity l_{12} and by rotation about the OX_1 axis through angle α_{12}. The rotation matrix for the transition from the $X_2Y_2Z_2$ system to the $X_1Y_1Z_1$ system has the form

$$M_{12} = \begin{Vmatrix} 1 \\ 0 \\ 0 \end{Vmatrix} \tag{11}$$

The transfer matrix has the form

$$L_{12} = \begin{Vmatrix} l_{12} \\ 0 \\ 0 \end{Vmatrix}. \tag{12}$$

The matrix equation for the transition to the $X_0Y_0Z_0$ system has the form

$$X_1 = M_{12}M_{23}[M_{34}M_{45}(M_{56} \cdot X_6 + L_{56}) + L_{34}] + L_{12}, \tag{13}$$

where

X_1 is the column of the coordinates of point 0 in the $X_1Y_1Z_1$ system.

The $X_0Y_0Z_0$ system can be combined with the $X_1Y_1Z_1$ system by rotation about the Z_0 axis angle γ_{01}. The rotation matrix for the transition from the $X_1Y_1Z_1$ system to the $X_0Y_0Z_0$ system has the form

(14)

The matrix equation for the transition to the $X_0Y_0Z_0$ system has the form /33

$$X_0 = M_{01}\{M_{12}M_{23}[M_{34}M_{45}(M_{56}\cdot X_6 + L_{56}) + L_{34}] + L_{12}\}. \quad (15)$$

Substituting the matrices into Eq. (15), and operating on them, we derive expressions linking the coordinates of the arbitrary point of the tongs link 6, to the $X_0Y_0Z_0$ and $X_6Y_6Z_6$ systems

$$\left.\begin{aligned} X_0 &= \cos\gamma_{01}(X_3\cos\gamma_{23} - Y_3\sin\gamma_{23} + l_{12}) - \sin\gamma_{01}\cos\alpha_{12}\times \\ &\times (X_3\sin\gamma_{23} + Y_3\cos\gamma_{23}) - Z_3\sin\gamma_{01}\sin\alpha_{12}; \\ Y_0 &= \sin\gamma_{01}(X_3\cos\gamma_{23} - Y_3\sin\gamma_{23} + l_{12}) + \cos\gamma_{01}\cdot\cos\alpha_{12}\times \\ &\times (X_3\sin\gamma_{23} + Y_3\cos\gamma_{23}) - Z_3\cos\gamma_{01}\sin\alpha_{12}; \\ Z_0 &= \sin\alpha_{12}(X_3\sin\gamma_{23} + Y_3\cos\gamma_{23}) + Z_3\cos\alpha_{12}, \end{aligned}\right\} \quad (16)$$

where

$$X_3 = \cos\gamma_{45}(X_6 + l_{56}) - \sin\gamma_{45}(Y$$

$$Y_3 = \cos\alpha_{34}\sin\gamma_{45}(X_6 + l$$
$$- Z_6\sin\alpha_{56}) - \sin\alpha_{34}$$

$$Z_3 = \sin\alpha_{34}\sin\gamma_{45}(X_6 + l_{56}) + \sin$$
$$+ \cos\alpha_{34}(Y_6\sin\alpha_{56} + Z_6\cdot\cos\alpha_{56}$$

The velocities and accelerations of arbitrary point O of the tongs link can be determined by differentiation of the expressions in (16) with respecto to time. The velocities and acclerations of tong link points obviously will depend on the velocities and accelerations of change in the mechanism's generalized coordinates. Let us now find the Euler angles fixing the position of the tongs link with the solid moved. The matrix of the direction cosines fixing the position of the $X_6Y_6Z_6$ system, which is rigidly connected to the tongs link, relative to the stationary coordinate system has the form

$$M_{60} = \begin{Vmatrix} m_{11}m_{12}m_{13} \\ m_{21}m_{22}m_{23} \\ m_{31}m_{32}m_{33} \end{Vmatrix}. \quad (17)$$

$$
\begin{aligned}
m_{11} &= \cos\varphi_{60}\cos\psi_{60} - \sin\varphi_{60}\cos\theta_{60}\sin\varphi_{60};\\
m_{12} &= -\sin\varphi_{60}\cos\psi_{60} - \cos\varphi_{60}\cos\theta_{60}\sin\psi_{60};\quad m_{13} = \sin\varphi_{60}\cdot\sin\theta_{60};\\
m_{21} &= -\sin\varphi_{60}\cdot\cos\psi_{60} - \cos\varphi_{60}\cos\theta_{60}\sin\psi_{60};\\
m_{22} &= -\sin\varphi_{60}\sin\psi_{60} + \cos\varphi_{60}\cdot\cos\theta_{60}\cdot\cos\psi_{60};\\
m_{23} &= \cos\varphi_{60}\cdot\sin\theta_{60};\quad m_{31} = \sin\psi_{60}\cdot\sin\theta_{60};\\
m_{32} &= -\cos\psi_{60}\sin\theta_{60};\quad m_{33} = \cos\theta_{60},
\end{aligned}
\tag{18}
$$

where

ψ_{60}, φ_{60}, θ_{60} are the Euler angles fixing the position of the $X_6Y_6Z_6$ system relative to the $X_0Y_0Z_0$ system.

Matrix (17) also can be derived by the method that transforms the vector given in the $X_6Y_6Z_6$ system into the $X_0Y_0Z_0$ system.

The matrix equation for the transformation has the form

$$
M_{01}M_{12}M_{23}M_{34}M_{45}M_{56}X_6 = M_{06}\cdot X_6 = X_0, \tag{19}
$$

where

X_0 and X_6 are, respectively, the columns of the coordinates of the vector in the $X_0Y_0Z_0$ and $X_6Y_6Z_6$ systems;

M_{06} is the matrix for the transition from the $X_6Y_6Z_6$ system to the $X_0Y_0Z_0$ system. /34

Eq. (19) yields

$$
M_{06} = M_{01}\cdot M_{12}\cdot M_{23}\cdot M_{34}\cdot M_{45}\cdot M_{56}. \tag{20}
$$

M_{60} and M_{06} are transposed matrices with respect to each other, so once M_{06} is determined, M_{60} is readily found.

Performing the multiplication of the matrices in the right-hand side of Eq. (20), and equating the elements of matrix (17) to transposed matrix M_{06}, we derive nine equations, from which it is easy to determine the Euler angles fixing the position of the tongs link in terms of the mechanism's generalized coordinates.

Of the nine equations, we shall use only the three needed to determine the Euler angles:

$$\cos\theta_{60} = \sin\alpha_{12}\, n_{23} + \cos\alpha_{12}\, n_{33};$$
$$-\cos\psi_{60}\sin\theta_{60} = \sin\gamma_{01}\, n_{13} + \cos\gamma_{01}(\cos\alpha_{12} n_{23} + \sin\alpha_{12}\, n_{33}); \qquad (21)$$
$$\cos\varphi_{60}\sin\theta_{60} = \sin\alpha_{12}\, n_{22} + \cos\alpha_{12}\, n_{32}.$$

$$u_{13} = \cos\gamma_{23}\sin\gamma_{45}\sin\alpha_{56} + \sin\gamma_{23}(\cos\alpha_{34}\cos\gamma_{45}\sin\alpha_{56} + \cos\alpha_{56} + \sin\alpha_{34});$$
$$u_{23} = \sin\gamma_{23}\sin\gamma_{45}\sin\alpha_{56} - \cos\gamma_{23}(\cos\alpha_{34}\cos\gamma_{45}\sin\alpha_{56} + \cos\alpha_{56}\sin\alpha_{34}); \qquad (22)$$
$$u_{33} = -\sin\alpha_{34}\cos\gamma_{45}\sin\alpha_{56} + \cos\alpha_{56}\cos\alpha_{34};$$
$$u_{22} = -\sin\gamma_{23}\sin\gamma_{45}\cos\alpha_{56} = \cos\gamma_{23}(\cos\alpha_{34}\cos\gamma_{45}\cos\alpha_{56} - \sin\alpha_{34}\sin\alpha_{56});$$
$$u_{32} = \sin\alpha_{34}\cos\gamma_{45}\cos\alpha_{56} + \sin\alpha_{56}\cos\alpha_{34}.$$

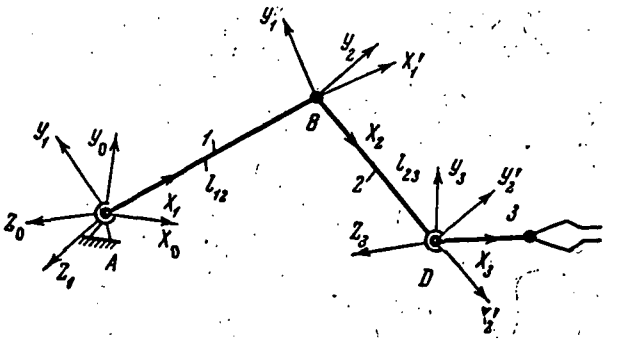

Figure 2.

Eqs. (21) and (22) provide the Euler angles and their derivatives with respect to time. Knowing the Euler angles and their derivatives, it is easy to use known formulas to find the projections of the angular velocity and the angular acceleration of link 6 on the fixed, $X_0Y_0Z_0$, and movable $X_6Y_6Z_6$, axes.

Let us next consider the kinematics of the spatial slave with spherical pairs shown in Figure 2 (a manipulator of the "Ruka" type [1]). This mechanism has seven degrees of freedom. Link 1 forms a 3d class spherical pair with the support, link 1 forms a 5th class turning pair with link 2, and link 2 forms a 3d class spherical pair with link 3.

Let us associate a coordinate system with the links in the mechanism as follows. Let us associate the $X_0Y_0Z_0$ system with the support, placing the origin at A. Let us associate the $X_1Y_1Z_1$ system with link 1, directing the X_1 axis along the axis of link 1, and the Z_2 axis parallel to the axis of turning pair B. The origin of the $X_1Y_1Z_1$ system too is at A. Let us associate the

$X_2Y_2Z_2$ system with link 2, placing the origin at B, and let us direct the X_2 axis along axis 2, and we direct the Z_2 axis along the axis of turning pair B. Let us associate the $X_3Y_3Z_3$ system with link 3, placing the origin at hinge D. Let us direct the X_3 axis along the axis of link 3, and the Z_3 axis parallel to the axis of rotation of the jaws of the tongs link. 35

Let arbitrary point O have coordinates $X_3Y_3Z_3$ in the $X_3Y_3Z_3$ system. Let us find the coordinates of this same point in the $X_0Y_0Z_0$ system.

The $X_2Y_2Z_2$ system can be combined with the $X_3Y_3Z_3$ system by transfer along the X_2 axis by the magnitude l_{23} and three successive turns through three Euler angles, ψ_{32}, θ_{32}, and φ_{32}.

The transfer matrix has the form

$$L_{23} = \begin{Vmatrix} l_{23} \\ 0 \\ 0 \end{Vmatrix}. \tag{23}$$

The rotation matrix for the transition from the $X_3Y_3Z_3$ system to the $X_2Y_2Z_2$ system can be derived by the transposition of the resultant matrix for the three turns through angles ψ_{32}, θ_{32}, and φ_{32}, which determine the position of the $X_3Y_3Z_3$ system relative to the $X_2Y_2Z_2$ system.

$$M_{23} = \begin{Vmatrix} n_{11} & n_{12} & n_{13} \\ n_{21} & n_{22} & n_{23} \\ n_{31} & n_{32} & n_{33} \end{Vmatrix}, \tag{24}$$

where

$$n_{11} = \cos\psi_{32}\cos\varphi_{32} - \sin\psi_{32}\cos\theta_{32}\sin\varphi_{32};$$
$$n_{12} = -\cos\psi_{32}\sin\varphi_{32} - \sin\psi_{32}\cos\theta_{32}\cos\psi_{32}; \quad n_{13} = \sin\psi_{32}\cdot\sin\theta_{32};$$
$$n_{21} = \sin\psi_{32}\cos\varphi + \cos\psi_{32}\cos\theta_{32}\sin\varphi_{32};$$
$$n_{22} = -\sin\psi_{32}\sin\varphi_{32} + \cos\psi_{32}\cos\theta_{32}\cos\varphi_{32};$$
$$n_{23} = -\cos\psi_{32}\sin\theta_{32}; \quad n_{31} = \sin\theta_{32}\sin\varphi_{32};$$
$$n_{33} = \sin\theta_{32}\cos\varphi_{32}; \quad n_{33} = \cos\theta_{33}.$$

The $X_1Y_1Z_1$ system can be combined with the $X_2Y_2Z_2$ system by parallel transfer along the X_1 axis by the magnitude l_{12} and rotation about the Z_1 axis through angle γ_{12}. The transfer matrix for the transfer from the $X_2Y_2Z_2$ system to the

$X_1Y_1Z_1$ system has the form

$$L_{12} = \begin{Vmatrix} l_{12} \\ 0 \\ 0 \end{Vmatrix}. \tag{25}$$

The rotation matrix for the transition from the $X_2Y_2Z_2$ system to the $X_1Y_1Z_1$ system has the form

$$M_{12} = \begin{Vmatrix} \cos\gamma_{12} & -\sin\gamma_{12} & 0 \\ \sin\gamma_{12} & \cos\gamma_{12} & 0 \\ 0 & 0 & 1 \end{Vmatrix}. \tag{26}$$

The $X_0Y_0Z_0$ system can be combined with the $X_1Y_1Z_1$ system by three successive turns through Euler angles $\psi_{10}\theta_{10}\varphi_{10}$. The matrix for the transition from the $X_1Y_1Z_1$ system to the $X_0Y_0Z_0$ system has the form

$$M_{01} = \begin{Vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{Vmatrix}, \tag{27}$$

where the matrix elements b_{ij} (i, j = 1,2,3) have a form similar to that of the matrix elements n_{ij} (i, j = 1,2,3) in Eq. (24), with the only difference being that the subscript "32" for angles ψ, θ, φ is replaced by the subscript "10". /36

The matrix equation for the transformation of the coordinates of the point given in the $X_3Y_3Z_3$ system to the $X_0Y_0Z_0$ system can be written in the form

$$X_0 = M_{01} M_{12} (M_{23} \cdot X_3 + L_2) + L_1, \tag{28}$$

where

X_0 and X_3 are, respectively, the columns of coordinates of an arbitrary point in the systems.

Operating on the matrices included in Eq. (28), we obtain

$$X_0 = b_{11}[\cos\gamma_{12}(X_3 n_{11} + Y_3 n_{12} + Z_3 n_{13} + l_{23}) = \sin\gamma_{12}(X_3 n_{21} + Y_3 n_{22} + Z_3 n_{33}) + l_{12}] + b_{12}[\sin\gamma_{12}(X_3 n_{11} + Y_3 n_{12} + Z_3 n_{13} + l_{23}) + \cos\gamma_{12}(X_3 n_{21} + Y_3 n_{22} + Z_3 n_{33})] + b_{13}(X_3 n_{31} + Y_3 n_{32} + Z_3 n_{33});$$

$$Y_0 = b_{21}[\cos\gamma_{12}(X_3 n_{11} + Y_3 n_{12} + Z_3 n_{13} + l_{23}) = \sin\gamma_{12}(X_3 n_{21} + Y_3 n_{22} + Z_3 n_{23}) + l_{12}] + b_{22}[\sin\gamma_{12}(X_3 n_{11} + Y_3 n_{12} + Z_3 n_{13} + l_{23}) + \cos\gamma_{12}(X_3 n_{21} + Y_3 n_{22} + Z_3 n_{23})] + b_{13}(X_3 n_{31} + Y_3 n_{32} + Z_3 n_{33}); \quad (29)$$

$$Z_0 = b_{31}[\cos\gamma_{12}(X_3 n_{11} + Y_3 n_{12} + Z_3 n_{13} + l_{23}) - \sin\gamma_{12}(X_3 n_{21} + Y_3 n_{22} + Z_3 n_{23}) + l_{12}] + b_{32}[\sin\gamma_{12}(X_3 n_{11} + Y_3 n_{12} + Z_3 n_{13} + l_{23}) + \cos\gamma_{12}(X_3 n_{21} + Y_3 n_{22} + Z_3 n_{23})] + b_{33}(X_3 n_{31} + Y_3 n_{32} + Z_3 n_{33}).$$

The velocities and accelerations of an arbitrary point in link 3 can be determined by the differentiation of Eq. (29) with respect to time.

Let us find the Euler angles ψ_{30}, φ_{30}, and θ_{30}, that fix the position of tongs link 3 relative to the $X_0Y_0Z_0$ system.

The matrix equation for the transformation of the coordinates of the vector from the $X_3Y_3Z_3$ system to the $X_0Y_0Z_0$ system has the form

$$X_0 = M_{03} \cdot X_3. \quad (30)$$

The equation for the transformation of the coordinates of the vector for the transition from the $X_3Y_3Z_3$ system to the $X_0Y_0Z_0$ system can be written in the following form by using the transition matrices M_{32}, M_{21}, and M_{01}

$$X_0 = M_{01} M_{12} M_{23} X_3 \cdot X_3. \quad (31)$$

Comparing Eqs. (30) and (31)

$$M_{03} = M_{01}M_{12}M_{23}. \quad (32)$$

Multiplying the matrices in the right-hand side, we derive the expressions for the elements in terms of known generalized coordinates.

The matrix of direction cosines, M_{30}, fixing the position of the $X_3Y_3Z_3$ system relative to the $X_0Y_0Z_0$ system is similar in form to Eq. (17), except that the subscript "60" for angles ψ, θ, and φ is replaced by "30".

Equating the elements of the M_{30} matrix and the transposed matrix, M_{03}, from Eq. (32), we derive nine equations for finding the Euler angles.

The three equations needed to solve the problem are /37

$$\cos\theta_{30} = b_{31}(\cos\gamma_{12}n_{13} - \sin\gamma_{12}n_{23}) + b_{32}(\sin\gamma_{12}n_{13} + \cos\gamma_{12}n_{23}) + b_{33}n_{33};$$

$$\sin\psi_{30}\sin\theta_{30} = b_{11}(\cos\gamma_{12}n_{13} - \sin\gamma_{12}n_{23}) + b_{12}(\sin\gamma_{12}n_{13} + \cos\gamma_{12}\cdot n_{23}) + b_{13}n_{33}; \quad (33)$$

$$\sin\varphi_{30}\sin\theta_{30} = b_{31}(\cos\gamma_{12}n_{11} - \sin\gamma_{12}n_{21}) + b_{32}(\sin\gamma_{12}\cdot n_{11} + \cos\gamma_{12}\cdot n_{21}) + b_{33}n_{31}.$$

The derivatives of the Euler angles with respect to time can be found by differentiating Eq. (33).

The angular velocities and accelerations of link 3 now can be found quite readily from known formulas.

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